MATHEMATICS

WAVELETS ON NONUNIFORM KNOT SEQUENCES

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A traditional wavelet basis generated by the shifts and dilations of a single wavelet can be considered to be centered on the knot sequences $\{a^j\}_{j\in\mathbf{Z}}$, where $a^j = \{i2^{-j}\}_{i\in\mathbf{Z}}$. These knot sequences possess both shift-invariant uniformity, that is, the points in each knot sequence a^j are uniformly spaced, and scale-invariant uniformity, that is, the knot sequence a^{j+1} can be constructed in a uniform manner from the knot sequence a^j. The research presented in this paper develops techniques for constructing generalized wavelet bases centered on nonuniform knot sequences.

A method is given to construct a nested sequence of spaces generated by orthogonal bases centered on nested knot sequences intertwined with a given nested sequence of spaces. If the given sequence is a multiresolution analysis, then the intertwined sequence of spaces is as well. The construction gives a sequence of refinement masks, and a method is given for constructing the corresponding wavelet masks. A particular type of basis centered on a knot sequence is considered, one generated by a macroelement, a set of functions supported on $[0, 1]$ satisfying certain properties.

The main results hold when each knot sequence is constructed by adding precisely one knot point between each pair of adjacent knot points in the prior knot sequence. Further results are presented in the semi-regular setting, in which knot sequences are generated through dyadic refinements of an initial nonuniform knot sequence. It is hoped that representing a given function in terms of a generalized wavelet basis centered on well-chosen knot sequences can yield a better representation of the function than in an equivalent uniform setting. A data compression example is included as evidence supporting this assertion.